Therefore, using D for $\frac{d}{dx}$, and *a' b* for *a or b*,

$$\lim \frac{u'u'' \ldots}{v'v' \ldots} = 0' \lim \left(\frac{D^{s'}u' D^{s''}u'' \ldots}{s'!\, s''! \ldots} \div \frac{D^{t'}v' D^{t''}v'' \ldots}{t'!\, t''! \ldots} \right)' \infty,$$

according as $s'+s''+\ldots >, =$ or $< t'+t''+\ldots$ (1)

A special case of this problem is to find, dropping the accents, limit $\frac{u^m}{v^n}$, where m and n are any quantities.

We have the identity $\lim \frac{u^m}{v^n} = \lim \left(\frac{u^t}{v^s}\right)^{\frac{n}{s}} \lim u^{\frac{ms-nt}{s}}$, and this is, if no derivative of u or v of not greater order than st is infinite,

$$0' \lim \left[\left(\frac{D^s u}{s!}\right)^m \div \left(\frac{D^t v}{t!}\right)^n \right]' \infty, \text{ according as } ms >, = \text{ or } < nt. \quad (2)$$

NOTE —We may find A inductively, using Leibnitz's Theeorem. Thus, when $a = a$,

$$D^{s'}u' = \frac{s'!}{s'!} D^{s'}u'$$

$$D^{s'+s''}u'u'' = \frac{(s'+s'') \ldots (s'+1)}{s''!} D^{s'}u' D^{s''}u'' = \frac{(s'+s'')!}{s'!\, s''!} D^{s'}u' D^{s''}u''$$

$$D^{s'+s''+s'''}u'u''u''' = \frac{(s'+s''+s''') \ldots (s'+s''+1)}{s'''!} D^{s'+s''}u'u'' D^{s'''}u'''$$

$$= \frac{(s'+s''+s''')!}{s'!\, s''!\, s'''!} D^{s'}u' D^{s''}u'' D^{s'''}u'''$$

. . . .

EXAMPLES.

$$\lim \frac{\sin^3(a-1)[2\cos \frac{1}{2}\pi a + \pi(a-1)][2\cos(a-1) - \cos^2(a-1) + \frac{1}{4}(a-1)^4]}{[(\log a)^2 - (a-1)^2 \div a][2\cos(1-a) + a^2 - 2a - 1][8\varepsilon^{a-1} - \varepsilon^{2a-2} - 2a(a+1) - 3]}$$

$$= (a=1)\ \tfrac{3}{4}\pi^3.$$

$$\frac{[a-1-\sin(a-1)]^{\frac{1}{2}}}{[1-\cos(a-1)]^{\frac{3}{4}}} = (a=1)\ [\tfrac{1}{3}\sqrt{2}]^{\frac{1}{2}};$$ Ex. 55, chap. 10, Tod. Dif. Cal.

DEMONSTRATION OF THE PROP. AT PAGE 8.

BY PROF. D. J. MC. ADAM, WASHINGTON, PA.

Let $B\,CD\,A$ be a semicircle, diameter AB, $ABCD$ being any inscribed trapezium, to prove that the sides AD, DC and CB may represent the reciprocals of the lines a, b, c if AB represent the reciprocal of r.

Draw AC. Then is

$$\begin{aligned} AB^2 &= BC^2+AC^2 = BC^2+CD^2+AD^2-2AD\times CD\cos D \\ &= BC^2+DC^2+AD^2+2AD\times DC\sin CAB \\ &= BC^2+DC^2+AD^2+2\,\frac{AD\times DC\times BC}{AB}. \qquad (1) \end{aligned}$$

In a triangle with the given notation,

$$\frac{1}{a^2}=\frac{\sin^2\frac{1}{2}A}{r^2};\ \frac{1}{b^2}=\frac{\sin^2\frac{1}{2}B}{r^2};\ \frac{1}{c^2}=\frac{\sin^2\frac{1}{2}C}{r^2};$$

$$\begin{aligned} \therefore\quad \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2} &= \frac{1}{r^2}\left(\sin^2\tfrac{1}{2}A+\sin^2\tfrac{1}{2}B+\sin^2\tfrac{1}{2}C\right) \\ &= \frac{1}{r^2}\left(1-2\sin\tfrac{1}{2}A\sin\tfrac{1}{2}B\sin\tfrac{1}{2}C\right) \\ &= \frac{1}{r^2}\left(1-\frac{2r^3}{abc}\right). \qquad (2) \end{aligned}$$

Substituting values of $\sin\frac{1}{2}A$ &c.,

$$\frac{1}{r^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{2r}{abc}. \qquad (3)$$

Comparing (1) with (3) we find $1\div r^2$ corresponds to AB^2, $1\div a^2$ to BC^2 &c., hence the values of these reciprocals might be made the sides of a trapezium npon AB.

[A demonstration of the abov prop. was also sent by Mr. E. B. Seitz.]

SOLUTIONS OF PROBLEMS IN NUMBER TWO.

SOLUTIONS of problems in No. 2 have been received as follows:

From Amateur, 199; R. J. Adcock, 200, 201; Marcus Baker, 196, 197; Prof. W. P. Casey, 196, 197, 200; Prof. P. E. Chase, 197, 200; Geo. M. Day, 196, 197; E. L. De Forest, 201; Capt. J. L. de Fremery, 196, 197; W. E. Heal, 196; H. Heaton, 196, 197, 198, 200; Prof. E. W. Hyde, 201; Chas. H. Kummell, 198, 201; Prof. J. H. Kershner, 196, 197; W. V. Mc. Knight, 196, 197; Prof. D. J. Mc. Adam, 196, 197, 200; Prof. H. T. J. Ludwick, 198; Artemas Martin, 196, 197, 198; K. S. Putnam, 196, 197; P. Richardson, 197; Prof. J. Scheffer, 196, 197, 199, 200; Prof. T. A. Smith, 196, 197; E. B. Seitz, 196, 197, 198, 200; Anna T. Snyder, 197; T. P. Stowell, 197; C. A. Van Velzer, 196, 197, 200.

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196. "The sides of a triangle are respectively x^2+x+1, $2x+1$ and x^2-1, x being any number greater than one; prove that the angle opposite the side x^2+x+1 is equal to $120°$."